## * Scalar Potential :—

We know that the workdone by the static electric field $\vec{E}$ when a test charge $q_0$ moves through a vector distance $\vec{dl}$ is given by

$$dW = q_0 \vec{E} \cdot \vec{dl} \quad \text{— (i)}$$

∴ The workdone in taking a test charge $q_0$ moves through a vector distance $\vec{dl}$ against the electric field is given by

$$dW = -q_0 \vec{E} \cdot \vec{dl}. \quad \text{— (ii)}$$

∴ The workdone in taking a unit positive test charge from infinity to a point P in the electric field is

$$\frac{dW}{q_0} = -\int_{\infty}^{P} \vec{E} \cdot \vec{dl} \quad \text{— (iii)}$$

which is equal to the potential V at the point

$$\therefore V = -\int_{\infty}^{P} \vec{E} \cdot \vec{dl} \quad \text{— (iv)}$$

But the R.H.S. of this expression is the negative line integral of the electric field. Hence, in other words, the electric potential is the negative line integral of the electric field. From the above equation, we may write as

$$dV = -\vec{E}\cdot\vec{dl} \quad \text{—— (V)}$$

If the co-ordinates of the point P are $x, y, z$ then $V(x,y,z)$ is a function of these co-ordinates. The partial derivatives $\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}$ represents the rate of change of V with x, y and z respectively.

The change dV in the potential produced due to a small displacement l with components dx, dy and dz along x, y and z axes respectively is given by

$$dV = \frac{\partial V}{\partial x}\cdot dx + \frac{\partial V}{\partial y}\cdot dy + \frac{\partial V}{\partial z}\cdot dz$$

$$= \left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)\cdot\left(dx\,\vec{i} + dy\cdot\vec{j} + dz\,\vec{k}\right)$$

$$= (\text{grad}\cdot V)\cdot\vec{dl}$$

$$\left[\because \vec{dl} = dx\,\vec{i} + dy\cdot\vec{j} + dz\cdot\vec{k}\right)$$

Comparing this with eqn (V), we get,

$$-\vec{E}\cdot\vec{dl} = -(\text{grad } V)\cdot\vec{dl} \; \cancel{= \nabla V \text{ —— (vi)}}$$

$$\Rightarrow \vec{E} = -\text{grad } V$$

$$= \nabla V \quad \text{—— (vi)}$$

This shows that the vector $\vec{E}$ is the gradient of the scalar function V. Such potential is called a Scalar potential.